

Practice exam 2

1. Determine whether the following statements are true or false. Explain why it is true or give an example that disproves the statement.

(a) If f has a local minimum at (a, b) and f is differentiable at (a, b) , then $\nabla f(a, b) = 0$.

(b) If f has two local maxima, then f must have a local minimum.

(c) If $|\gamma(t)| = 1$ for all t , then $|\gamma'(t)|$ is constant.

(d) If $|\gamma(t)| = 1$ for all t , then $\gamma'(t)$ is orthogonal to $\gamma(t)$ for all t .

2. Find the volume of the solid region between the surfaces $z = 2x^2 + 2y^2$ and $z = 12 - x^2 - y^2$.

3. Evaluate the iterated integral

$$\int_0^1 \int_x^1 \frac{\cos y}{y} dy dx.$$

4. Find the maximum rate of change of $f(x, y) = x^2y + \sqrt{y}$ at the point $(2, 1)$. In which direction does it occur?

5. The helix $\gamma_1(t) = (\cos t, \sin t, t)$ intersects the curve $\gamma_2(t) = (1+t, t^2, t^3)$ at the point $(1, 0, 0)$. Find the angle of intersection of these curves. (Remember that the angle of intersection of two curves is the angle of their tangent lines at the intersection point).

6. Find the distance from the origin to the line $x = 1 + t$, $y = 2 - t$, $z = -1 + 2t$.

7. Find the absolute maximum and minimum values of f on the disk $x^2 + y^2 \leq 4$.

$$f(x, y) = e^{-x^2-y^2}(x^2 + 2y^2).$$