

Practice exam 1

1. Determine if the following statements are true (T) or false (F). *You don't need to justify your answer.* (10 pts)

(a) If f is a function of two variables, then ∇f is perpendicular to the graph of f .

(b) If $u, v \in \mathbb{R}^3$ and $u \times v = v \times u$, then u and v lie on the same line.

(c) $\lim_{(x,y) \rightarrow (0,0)} \frac{x \sin y}{\sqrt{x^2 + y^2}} = 0$.

(d) If f and g are discontinuous, then $f + g$ is discontinuous.

(e) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^3}{x^2 + y^6} = 0$.

2. (a) Find a parametrization for the curve which is the intersection of the cylinder $x^2 + y^2 = 16$ and the plane $x + z = 5$. Make sure you don't go around the curve more than once. (6 pts)

(Hint: don't try to set one of the variables to be the parameter)

(b) Compute the acceleration vector at $t = 0$. (4 pts)

3. Find an equation for the plane through $(1, 2, -2)$ that contains the line

$$x = 2t, \quad y = 3 - t, \quad z = 1 + 3t.$$

(10 pts)

4. Find the critical points of f and classify all of them.

$$f(x, y) = x^3 - 6xy + 8y^3.$$

(10 pts)

5. Find the points on the surface $xy^2z^3 = 6$ that are the closest to the origin. (10 pts)

6. Find all the points on the hyperboloid $x^2 + 4y^2 - z^2 = 4$ where the tangent plane is parallel to the plane $2x + 2y + z = 5$. (10 pts)